II Semester M.Sc. Degree Examination, June/July 2014
(RNS) (2011-12 & Onwards)
MATHEMATICS
M-204 : Partial Differential Equations

Time : 3 Hours Max. Marks : 80

Instructions : *i) Answer **any five** (**5**) **full** questions choosing atleast (**2**) **two** from **each** Part.*
*ii) **All** questions carry **equal** marks.*

PART – A

1. a) Define the Cauchy's problem for first order partial differential equations and hence explain the method of characteristics for solving it. **6**

b) Solve $xu_x + (x + y) u_y = u + 1$ with $u (x, y) = x^2$ when $y = 0$. **5**

c) Solve $(x + 1) u_x + y u_y = 0$ with $u (0, y) = e^{-y}$. **5**

2. a) Solve : $u_t + uu_x = 0$ with

$$u(x, 0) = \begin{cases} 1 & , \quad x \le 0 \\ 1 - x, & \quad 0 \le x \le 1 \\ 0 & , \quad x \ge 1 \end{cases}$$ **6**

b) Solve $(u^2 - y^2) u_x + xyu_y + xu = 0$ with $u = y = x$, $x > 0$. **5**

c) Solve $p^2 - 3q^2 - u = 0$ with $u (x, 0) = x^2$. **5**

3. a) Solve : $u_{xxx} + u_{yyy} - u_{xyy} - u_{yyy} = e^{2x + y} + \sin (x + 2y)$. **6**

b) Classify the equation. $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ and hence reduce it into its canonical forms. **5**

c) Reduce the equation
$xyr - (x^2 - y^2)s - xyt + yp - xq = 2 (x^2 + y^2)$ into its canonical forms. **5**

P.T.O.

4. a) Explain the method of Monge's for solving the equation of the form
   Rr + Ss + Tt = V with usual notations. **6**

   b) Solve by Monge's method $(Hq)^2 r - 2(1 + p + q + pq)s + (1 + p)^2 t = 0$. **5**

   c) Solve $\left(D^3 + D^2D^1 - DD^{1^2} - D^{1^3}\right)z = ex\cos 2y$. **5**

PART – B

5. a) Solve by using appropriate Fourier transform the following IBVP :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, 0 \le t \le \infty$$

   subject to

$$\left.\begin{array}{l} u(x, 0) = f(x) \\ \dfrac{\partial u}{\partial t}(x, 0) = 0 \end{array}\right\}, -\infty < x < \infty$$

   8

   b) Show that a variable separable solution of the wave equation in spherical polar co-ordinates give rise to a legendre differential equation. **8**

6. Solve the Dirichlet and Churchill problems involving the Laplace equation in the upper-halfplane. **16**

7. a) Solve by using appropriate Fourier transform the following IBVP :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; 0 \le x, t < \infty$$

   subject to

   $u(x, 0) = f(x), \quad x \ge 0$

   $u(0, t) = 0 \quad, \quad t \ge 0.$ **8**

   b) Show that a variable separable solution of the diffusion equation in cylindrical polar co-ordinates gives rise to a Bessel differential equation. **8**

8. a) Using a simple example involving a hyperbolic or elliptic or parabolic equation illustrate Green's function method. **8**

   b) By making use of a successive approximation method obtain the solution of $xu_x + u_y = u^2$ subject to $u(x, 0) = x^2$, $u(0, y) = 0$. **8**
